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## **Remark to the Comment on "New pseudoclassical model for Weyl particles"**

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### **Abstract**

We present here our considerations concerning the problem of classical consistency of pseudoclassical models touched upon in a recent comment on our paper "New pseudoclassical model for Weyl particle".

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In a recent paper [1] we have proposed a pseudoclassical action to describe massless spin one-half particle in 3+1 dimensions. Quantization of this action reproduces the minimal theory of Weyl particle. The action has the form

$$S = \int_0^1 \left[ -\frac{1}{2e} \left( \dot{x}^\mu - i\psi^\mu \chi + i\varepsilon^{\mu\nu\rho\zeta} b_\nu \psi_\rho \psi_\zeta + \frac{\alpha}{2} b^\mu \right)^2 - i\psi_\mu \dot{\psi}^\mu \right] d\tau, \quad (1)$$

where  $x$ ,  $e, b$ , are even and  $\psi$ ,  $\chi$  odd (Grassmannian) variables, whereas  $\alpha$  is an even constant.

In a comment on our work [2] it was remarked that the classical theory, which corresponds to the action (1), is inconsistent if one regards  $\alpha$  as a real number. In spite of the fact that we did not consider  $\alpha$  in that sense at the classical level we have to assign only two real possible values  $\alpha = \pm 1$  to it at the quantum level. It is likely that it is this circumstance that was interpreted by the authors of the Comment as a problem or, as they claim, as an inconsistency of the model at the classical level. One ought to say that it is not for the first time that such a problem appears in the pseudoclassical mechanics (see, for example, [3]) and is certainly familiar to us. There exist different points of view on this problem. One of them was advocated before by the authors of the Comment [4] and implies, in fact, that some classical constants (in the case under consideration  $\alpha$ ) should be replaced by "dynamical variables". In the Comment the authors offer to modify our model [1] and the subsequent analogous models [5] in the same manner. Such a modification is possible indeed and we knew this way of action from the above mentioned publications. However, our point of view is that such a modification is not necessary, and moreover may be not relevant. The motivation is the following. It is known from numerous examples, very often some restrictions for possible values of parameters arise in the quantum theory. Such restrictions can in turn depend on details of quantization (in particular, in [2],  $\alpha = \pm 1$ , or  $\alpha = \pm 2$ , depending on the operators ordering). Thus, the quantum dynamics may impose restrictions on possible parameter values within the anticipated domain of their variation. We believe that one has to admit the same situation for the classical theory, and, moreover, to admit an alteration of the parameters nature in course of transition from the classical to the quantum theory (we do admit such an alteration for dynamical variables). In the model (1), at the classical level,  $\alpha$  has to be an even constant, of bifermionic type. In course of quantization it becomes into a real constant, whose possible values are defined by the quantum dynamics. Probably, such a treatment of the parameters in the pseudoclassical theory could be grounded if one might give a clear meaning to the pseudoclassical theory by constructing a "pseudoclassical" limit of the quantum theory.

According to another point of view we write the pseudoclassical action for purposes of the quantization only, then  $\alpha$  can be regarded from the beginning as a real number. That was done in our recent papers [5].

Also, following the ideas of Schwinger, we can probably consider from the very beginning all the variables entering in the action (1) as operators, and construct their commutation relations by means of the Schwinger-type action principle.

In any case, the point of view according to which the nature of parameters in pseudoclassical and quantum theory can be different, is until now acceptable and probably relevant to a "pseudoclassical" limit. In this respect the possible modifications proposed in the Comment have to be motivated by more essential reasons.

## References

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